METU FOOTBRIDGE: CABLE FORCE DETERMINATION AND DYNAMIC PROPERTIES

W. Njomo Wandji\textsuperscript{1} and A. Turer\textsuperscript{2}

ABSTRACT

Cable stayed bridges have gained preference amongst others for design and construction of pedestrian bridges in Turkey as a part of recent trend in engineering practice, as they can span larger distances and are light weight compared to commonly used beam type pedestrian bridges. However, various loads acting on cables may vary over time from the construction time until decommission. Dynamic forces, cable connectors or additional loads can affect the cables and safety of a cable stayed pedestrian bridge. To investigate the evolution of internal forces over the course of years, METU footbridge has been selected as a study case. Three different methods of various sophistication levels have been used to determine axial forces on cables at four different years and tracked their overall changes over the course of last 10 years since its construction in 2003. Influence of large sized advertising panels, which were asymmetrically added on the bridge’s sides by the municipality, has also been analyzed based on these measurements. Finite element modeling along with dynamic tests reached conclusions on long term behavior and effect of advertisement. Rain-wind induced vibration was also checked for individual cable stability. Out of this study, it has been found that guitar string and fixed-end beam theories lead to similar results, while afore-mentioned yielded larger forces. The recently added advertising panels significantly changed the distribution of loads in cables. Although rain-wind induced vibrations were found to be not generally critical, results shown that bridge stiffness is in a decreasing trend with time.

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Cable stayed bridges have gained preference amongst others for design and construction of pedestrian bridges in Turkey as a part of recent trend in engineering practice, as they can span larger distances and are light weight compared to commonly used beam type pedestrian bridges. However, various loads acting on cables may vary over time from the construction time until decommission. Dynamic forces, cable connectors or additional loads can affect the cables and safety of a cable stayed pedestrian bridge. To investigate the evolution of internal forces over the course of years, METU footbridge has been selected as a study case. Three different methods of various sophistication levels have been used to determine axial forces on cables at four different years and tracked their overall changes over the course of last 10 years since its construction in 2003. Influence of large sized advertising panels, which were asymmetrically added on the bridge’s sides by the municipality, has also been analyzed based on these measurements. Finite element modeling along with dynamic tests reached conclusions on long term behavior and effect of advertisement. Rain-wind induced vibration was also checked for individual cable stability. Out of this study, it has been found that guitar string and fixed-end beam theories lead to similar results, while afore-mentioned yielded larger forces. The recently added advertising panels significantly changed the distribution of loads in cables. Although rain-wind induced vibrations were found to be not generally critical, results shown that bridge stiffness is in a decreasing trend with time.

Introduction

METU Bridge is a pedestrian bridge located across Eskisehir Yolu (Dumlupinar Boulevard) as shown in Fig. 1. It is a cable-stayed bridge with a metal deck sustained by 12 cables (six on either side). This deck, which lays on two concrete abutments, is covered by tiles resting on thick steel slab which sandwich a mesh reinforced mortar. The cables are made of solid steel and attached to a back tilted pylon. METU Bridge is 48.45 m long between bearing spots, and 3.00 m wide between parapets. Figs. 2 and 3 show details of the bridge. The mechanical and physical properties of the steel footbridge are listed in Table 1.

METU Bridge has been erected in 2003. In 2011, municipal authorities placed an advertising board towards Ankara (East) side. Later, in 2012, another advertising panel, different in length from the previous one was placed on the opposite Eskisehir (western) side. The first of these panels is visible in Fig. 2a. The sequential and asymmetric loading on the bridge, in addition to the long-term effect of permanent loads coupled with dynamic live (exploitation, wind, temperature) loads acting on the deck have led forces along cables that vary significantly over the course of years.

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Figure 1. Location of METU Bridge.

Figure 2. Photographs of METU Bridge.

a) Side view  

b) Under view and southward abutment  

c) Northward abutment

Figure 3. METU Bridge dimensions.
Table 1. Mechanical and physical properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Structural parts</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steel</td>
<td>Cable</td>
<td></td>
</tr>
<tr>
<td>Mass Per Unit Volume</td>
<td>7.85E-09 N/mm³</td>
<td>7.85E-09 N/mm³</td>
<td></td>
</tr>
<tr>
<td>Weight Per Unit Volume</td>
<td>7.70E-05 N/mm³</td>
<td>7.70E-05 N/mm³</td>
<td></td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>2.10E+05 N/mm²</td>
<td>2.15E+05 N/mm²</td>
<td></td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>8.08E+04 N/mm²</td>
<td>8.28E+04 N/mm²</td>
<td></td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.30</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Coeff. of Thermal Expansion</td>
<td>1.17E-05</td>
<td>1.17E-05</td>
<td></td>
</tr>
</tbody>
</table>

The aim of this paper is to investigate the evolution of these forces over time. For this purpose, measurements have been carried out in four key years on the bridge, and three various methods have been used to determine the actual forces. In addition, rain-wind induced vibrations have been also studied in order to check the current cable stability, as it has evolved over time. Finally, an updated analytical model is built, so that it can serve as the basis of the study of human induced vibrations and fatigue analysis.

Dynamic testing: measurement and data processing

Measurement and data processing

From 2005 till today, four campaigns (2005, 2011, 2012, and 2013) of dynamic tests have been conducted on METU Bridge. Measurements utilized wireless triaxial accelerations measuring dynamic response of all twelve cables (Fig. 4) and the deck. They have been carried out under the Ambient Vibration Monitoring (AVM). A mobile notebook computer was used to operate wireless data acquisition system (DAS) to operate accelerometers and collect data. Dynamic measurements have been conducted with a sampling frequency of 256 Hz and 10 000 sweeps. In each campaign, measurements on the cables have been repeated a couple of times in order to capture the best estimate of their respective behavior. Then, in order to obtain the first few vibration modes of the deck, measurement points have been smartly chosen on the deck: they have been selected away from mid-span to capture at least the first and second mode shapes and away from the central longitudinal axis to capture torsional modes, as shown in Fig. 4.

Figure 4. Sensor locations on the deck. Figure 5. Absolute FFT values for cable 1.
The data processing conducted here has been carried out using Matlab platform [1]. For either deck or cables, data of each channel have been plotted in frequency domain using Fast Fourier Transform (FFT) [2]. For each cable, channels of all measurements have been plotted on the same graph. Absolute FFT values were good enough to successfully identify peak frequencies corresponding to the regularly repeating modes. Fig. 5 illustrates the graphical representation of the obtained data for cable #1 for the year 2013 as an example. Both absolute FFT values and imaginary parts are necessary for the deck in order to obtain natural vibration frequencies and mode shapes [2]. Fig. 6 shows peaks for the western node; dynamic behavior of the deck was considered independent of the cable vibration to obtain axial forces in cables, but was of great interest for structural identification.

![Graphs](image1)

Figure 6. Imaginary and absolute FFT values for West node of the deck.

**Cable force determination**

Three methods of different levels of sophistication have been used to determine axial force along cables. The first one is based on guitar string theory [2]. The tensile axial force \( T \) (in N) along a cable can be obtained through the formula:

\[
T = \frac{4mu^2 f_n^2}{n^2}
\]

where \( m \) (= 14.10 kg/m) is the linear mass density in kg/m, \( L \) is the cable length between anchorages, \( f_n \) is the frequency in Hz corresponding to mode order \( n \). However, this formulation does not account for many complexities that actually existed such as cable bending stiffness, material’s modulus of elasticity, cross-sectional area, moment of inertia, and boundary conditions; all that are to be considered especially for higher order modes. In this
perspective, Hiroshi Zui et al [3] developed a couple of relations using the modulus of elasticity, $E$; the cross-sectional area, $A$ ($= 1.735E^{-3}$ m$^2$); and the moment of inertia, $I$ ($= 2.39531E^{-3}$ m$^4$) as presented in Eq. 2.

1-In the case of using the natural frequency of first-order mode (cable with sufficiently small sag $3 \leq \Gamma$)

$$T = \begin{cases} 
  a_1(0.828 - 10.5a_2^2), & 0 \leq \xi \leq 6 \\
  a_1(0.865 - 11.6a_2^2), & 6 \leq \xi \leq 17 \\
  a_1(1 - 2.20a_2 - 0.550a_2^2), & 17 \leq \xi 
\end{cases}$$

(2a)

2- In the case of using the natural frequency of second-order mode (cable with sufficiently small sag $3 \geq \Gamma$)

$$T = \begin{cases} 
  a_1(0.882 - 85.0a_2^2), & 0 \leq \xi \leq 17 \\
  a_1(1.03 - 6.33a_2 - 1.58a_2^2), & 17 \leq \xi \leq 60 \\
  a_1(1 - 4.40a_2 - 1.10a_2^2), & 60 \leq \xi 
\end{cases}$$

(2b)

3-In the case of using the natural frequency of high-order modes (very long cable $2 \leq \Gamma$)

$$T = a_1(1 - 2.20n a_2), \quad 200 \leq \xi$$

(2c)

$$a_1 = \frac{4m}{n^2}; \quad a_2 = \frac{\sqrt{EI/m}}{fnL^2}; \quad \xi = L \sqrt{\frac{T}{EI}}$$

Furthermore, in 2005, Wenzel H. and Pichler D. [4] proposed another principle that is derived based on real beam theory and considering the end as fixed:

$$f_n = \frac{n}{2L} \left( \frac{T}{m} \right)^{\frac{1}{2}} \left[ 1 + \frac{2}{\eta} + \left( 4 + \frac{n^2 \pi^2}{2} \right) \times \frac{1}{\eta^2} \right]$$

with $\eta = L \left( \frac{T}{am} \right)^{\frac{1}{2}}$

(3)

These three methods have been used to determiate cable axial force for each of the years under study.

**Results of the Year 2005: Cable Tensile Force**

This corresponds to early age of the bridge just about after its construction. The application of the three force determination methods yielded the following results which are summarized in Fig. 7. In this figure, Method 1, Method 2, and Method 3 denote tensile forces obtained by Eq. 1, Eq. 2, and Eq. 3, respectively. Data collected in set [5] did not provide cable forces for cables 10, 11 and 12.

**Results of the Year 2011: Cable Frequencies and their Tensile Force**

Eight years after the construction of the bridge, and just before the installation of the first panel, experiments have been conducted again. Axial forces in the cables have then been determined as shown in Fig. 8.

**Results of the Year 2012: Cable Frequencies and their Tensile Force**

In 2012, one side (Eastern side) advertisement panel was installed. Following this event,
additional dynamic measurements have been conducted and results in terms of cable axial forces are presented in Fig. 9.

**Results of the Year 2013: Cable Frequencies and their Tensile Force**

In the year 2013, a second advertising panel has been installed on the western side in addition to the first one that was already installed on the Eastern side. The following chart in Fig. 10 illustrates the respective results obtained by using three respective methods referring to tensile forces obtained by Eq. 1, Eq. 2, and Eq. 3, respectively.

**Comparison and Discussions**

Several results can be drawn from the observation of the above graphs:

- Method 2 consistently yields lower values than Methods 1 and 3; Method 3 results in approximately similar values as Method 1. An important fact between the Method 2 and others is that it has different formulations based on the mode number; for example, if mode number is one, two or higher.

- Fig. 11 shows the evolution of cables’ forces as a function of tests conducted on different years. An important observation indicates that cable forces have decreased from 2005 to 2011, in cables 1, 2, 3, 4, 5, 7 and 9, while cables forces are increased in cables 6 and 8 (Fig 4).

- Naturally, in general, forces increase in cables with respect to a previous state when a panel weight is added. However, only cables 1 and 3 have obtained to have their forces slightly decreased from 2011 to 2012 when the first advertisement panel was added between these measurements. Similarly, cables 10 and 12 experienced the same phenomenon from 2012 and 2013 when the second panel was added on the western side. This might be a complicated affect due to nonlinearity or thermal effects.
Analytical Model and Structural Identification

Model updating

Based on dynamic tests conducted on METU Bridge, structural identification (St-Id) has been carried out on a finite element model built in the commercial software package SAP2000 [7]. The analytical model of the last configuration (with two panels) was prepared with some necessary assumptions that were made regarding various parameters, such as, mass density of the hypothetical constitutive material of the panels, panels’ elastic modulus, the equivalent thickness of the panels, the properties of links connecting panels to the rest of the structure, springs serving as supports between the deck and panels. Those parameters are iteratively modified so that updated analytical results became similar to the experimental ones.

- **Mass density of panel material**: panel frame constitutive material has been assumed to be an alloy of type aluminum 6063 whose density may vary between ±2%. Therefore,
a panel mass density of 53.122 kg/m³ has been obtained after iterations starting from 54.16 kg/m³. It is noteworthy to underline that the geometric dimensions used in mass density is the circumscribed dimensions that can just envelop the panels as can be seen in Fig. 13.

- **Elastic modulus of panel material**: also here for the same material, 67 620 MPa instead of 69 000 MPa led to better comparison with the experimental results.

- **Stiffness contribution of panels**: for updating process, it has been considered that the panels do not significantly participate to the stiffness of the bridge, but contribute by their masses. Therefore, their stiffness values have been reduced to one per cent of the conservatively computed plate values by modifying their moment of inertia.

- **Thickness of panels**: initially suggested to be 50 cm, the equivalent thickness has finally been chosen as 30 cm after calibration process.

- **Panel links**: after many iterations, panel links have been modeled with stiffness values 750 000 N/m for all translational directions and 750 000 N/rad for all rotational ones.

- **Deck supports**: spring properties have been finally set to those shown in Table 3 after many iterations. Z1 and Z2 are supports placed on METU side (south edge, pylon side), while Z3 and Z4 are located in the opposite side (northern side).

Table 3. Deck support spring stiffness values.

<table>
<thead>
<tr>
<th>Supports</th>
<th>UX (N/m)</th>
<th>UY (N/m)</th>
<th>UZ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KZ1</td>
<td>1.47E+08</td>
<td>1.47E+08</td>
<td>1.47E+08</td>
</tr>
<tr>
<td>KZ2</td>
<td>9.81E+07</td>
<td>9.81E+07</td>
<td>9.81E+07</td>
</tr>
<tr>
<td>KZ3</td>
<td>4903325</td>
<td>4903325</td>
<td>1.47E+08</td>
</tr>
<tr>
<td>KZ4</td>
<td>4412993</td>
<td>4412993</td>
<td>9.81E+07</td>
</tr>
</tbody>
</table>

Analytical results are now acquired from modal analysis. Table 4 compares experimental results versus those from finite element analysis. In Fig. 12, a graphical comparison between frequencies is shown as well as some of the deck mode shapes discussed in Table 4.

Table 4. Comparison of updated analytical and experimental frequencies of the deck.

<table>
<thead>
<tr>
<th></th>
<th>Analytical frequency (Hz)</th>
<th>Experimental frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st vertical bending mode</td>
<td>3.053</td>
<td>2.867</td>
</tr>
<tr>
<td>1st transversal bending mode</td>
<td>4.373</td>
<td>4.070</td>
</tr>
<tr>
<td>2nd vertical bending mode</td>
<td>5.020</td>
<td>4.531</td>
</tr>
<tr>
<td>1st torsional mode</td>
<td>6.271</td>
<td>6.236</td>
</tr>
<tr>
<td>3rd vertical bending mode</td>
<td>7.216</td>
<td>8.576</td>
</tr>
</tbody>
</table>

**Assessment of advertisement panels’ masses**

Two methods have been used to determined Advertisement panel’s masses. On one hand, the equivalent circumscribed volume after St-Id is 74.25 m³. Considering the density of 53.122 kg/m³ obtained in Section 4, it can be obtained a mass of 3944.309 kg. On the other hand, this
mass can be obtained by watching the evolution of forces applied on the deck. Indeed, the deck’s weight can be gotten from the updated structure by adding reactions at supports and cables’ forces. With an initial deck weight of 935.210 kN in 2005, the variation of deck’s weight between 2005 and 2013 is 38.502 kN that corresponds to 3924.771 kg. Considering that the eastern panel has a length of 42.9 m and the western panel 39.6 m, it can be deduced that the eastern panel’s mass is \( \Delta m_e = 2045.96 \) kg and the western panel’s mass is \( \Delta m_w = 1888.58 \) kg for a total mass of \( \Delta m_{ew} = 3934.54 \) kg.

### Deck’s stiffness

From the measurement of the deck, Table 5 summarizes natural frequencies of the ensemble formed by the deck and the panels according to their number. Considering only the first modes assuming to be dominant, frequencies can be expressed as \( \omega = \sqrt{k/(m_d + \Delta m)} \), where \( k \) is the ensemble stiffness, and \( \Delta m \) is the corresponding mass of the added panels. From this point, the assembly stiffness can be approximately computed by \( k = (m_d + \Delta m)\omega^2 \). Table 5 shows the stiffness values after computations. It can be remarked that whether vertical bending or torsion is considered, the deck stiffness decreases over time.

### Table 5. Deck’s natural frequencies (in rad/sec) and stiffness (in kN/m or kN/rad).

<table>
<thead>
<tr>
<th></th>
<th>2005</th>
<th>2011</th>
<th>2012 one side panel</th>
<th>2013 two side panels</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No panel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First</td>
<td>3.1 rad/sec</td>
<td>3.05 rad/sec</td>
<td>2.97 rad/sec</td>
<td>2.87 rad/sec</td>
</tr>
<tr>
<td>Second</td>
<td>4.8 rad/sec</td>
<td>4.88 rad/sec</td>
<td>4.50 rad/sec</td>
<td>4.53 rad/sec</td>
</tr>
<tr>
<td>Third</td>
<td>9.2 rad/sec</td>
<td>-</td>
<td>8.43 rad/sec</td>
<td>8.58 rad/sec</td>
</tr>
<tr>
<td>Fourth</td>
<td>12.1 rad/sec</td>
<td>-</td>
<td>12.14 rad/sec</td>
<td>-</td>
</tr>
<tr>
<td><strong>Vertical bending mode</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torsional bending mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Stiffness</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending</td>
<td>916 kN/m</td>
<td>887 kN/m</td>
<td>859 kN/m</td>
<td>816 kN/m</td>
</tr>
<tr>
<td>Torsion</td>
<td>4028 kN/rad</td>
<td>4028 kN/rad</td>
<td>4014 kN/rad</td>
<td>3860 kN/rad</td>
</tr>
</tbody>
</table>
Conclusions

This paper deals with cable force determination in a cable-stayed footbridge. The study is based on METU bridge by means of ambient vibration monitoring. Three different methods have been used to compute tensile cable force from cable geometric, physical, mechanical properties and the natural frequencies. Some modeling characteristics like boundary conditions or coupler’s presence are not considered. It has been seen that Method 2 yields to smaller values compared to the other methods. In addition, forces in cables were not changing monotonically: they could decrease in some cables even if mass due to advertisement panel was added. Indeed, cables 1 to 5 (away from the pylon) have their axial load decreasing between 2005 and 2011 though no change is bought to the bridge. On the other hand, cables 6 and 8 (middle of the bridge) have their loads increased during the same period, which is thought to be due to redistribution of forces between cables or between the cables and bending action of the deck. With the addition of the first and second panels, axial loads in the cables which are closer to the pylon are significantly increased, as much as two times while cables farther away from the pylon remained less sensitive to this modification. This behavior was attributed to the fact that longer cables had larger sag while shorter cables were closer to vertical and had smaller sag.

A stability check based on Scruton Number shows that some cables were potentially unstable. Unfortunately, lack of continuous dynamic monitoring data prevents to watch the stability status throughout the entire lifespan period. Another conclusion of the study was the continuous decay of the deck vertical stiffness although it would be expected that a slight increase would occur due to advertisement panels. An updated structural model has been proposed based on recorded data. The calibrated numerical model can be efficiently used as a basis of the study of human induced vibrations and fatigue analysis. Overall, reliability and long-term behavior of footbridges should consider possible stiffness and mass changes as they would influence both dynamic characteristics and redistribution of forces.

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References