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EQUIVALENT DAMPING FORMULATION FOR LRBs TO BE USED IN SIMPLIFIED ANALYSIS OF ISOLATED STRUCTURES

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ABSTRACT

In this paper, a new formulation to calculate the equivalent damping ratio for equivalent linear analysis of seismic isolated structures, specifically isolated with lead rubber bearings (LRBs), is proposed. Proposed formulation is capable of incorporating the variation in hysteretic behavior of LRBs due to reduction in strength of bearing as a function of lead core heating. To quantify the accuracy of the proposed equation in predicting the maximum isolator displacements (MIDs), MIDs obtained from a number of nonlinear time history analyses (NTHA) are compared with the predicted ones. NTHA are conducted with near field ground motions that constitute similar characteristics in terms of magnitude, closest distance to fault rupture, and local soil condition. In the analyses, effects of isolator properties namely, isolation period and characteristic strength on the efficiency of proposed formulation are also investigated. It is found that the proposed equation results in highly accurate estimations for MIDs, regardless of isolator characteristics.

Equivalent Damping Formulation For LRBs To Be Used In Simplified Analysis Of Isolated Structures

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ABSTRACT

In this paper, a new formulation to calculate the equivalent damping ratio for equivalent linear analysis of seismic isolated structures, specifically isolated with lead rubber bearings (LRBs), is proposed. Proposed formulation is capable of incorporating the variation in hysteretic behavior of LRBs due to reduction in strength of bearing as a function of lead core heating. To quantify the accuracy of the proposed equation in predicting the maximum isolator displacements (MIDs), MIDs obtained from a number of nonlinear time history analyses (NTHA) are compared with the predicted ones. NTHA are conducted with near field ground motions that constitute similar characteristics in terms of magnitude, closest distance to fault rupture, and local soil condition. In the analyses, effects of isolator properties namely, isolation period and characteristic strength on the efficiency of proposed formulation are also investigated. It is found that the proposed equation results in highly accurate estimations for MIDs, regardless of isolator characteristics.

Introduction

Due to its simplicity in calculation, one of the most common ways to determine the maximum isolator displacement (MID) of a seismically isolated structure (SIS) is a simplified method based on the use of elastic response spectrum. This method enables the prediction of maximum elastic deformations of a system with a representative period and damping ratio. However, since the behavior of isolators is nonlinear in nature, there is a need to identify an equivalent linear system which is able to represent the nonlinear characteristic of seismic isolation systems. Such linearization requires the definition of equivalent linear elastic terms namely, effective stiffness, k_{eff}, and equivalent damping ratio, ξ_{eq}. Since, it is an approximate solution, the accuracy of the simplified method of analysis in determining the response quantities of SIS has been tested by numerous studies in literature [1-6]. In these studies, accuracy of simplified method of analysis was investigated by comparing the estimated response quantities with the ones obtained from nonlinear time history analyses (NTHA). However, none of the previous studies considered the deterioration in force-deformation relation of isolators by assuming a non-deteriorating force-deformation relation for investigated isolators.

Recently published two companion papers revealed that there occurs a strength deterioration in hysteretic force-deformation behavior of a lead rubber bearing (LRB), which is among the most widely used isolators, under cyclic motion due to heat generated at the lead core [7,8]. Furthermore, recently, it has been revealed that there may be severe differentiation in MIDs of cases where lead core heating (LRBs are represented by a deteriorating hysteretic behavior) is considered and cases where non-deteriorating representation of hysteretic force-deformation of LRBs is employed [9-11]. Results of Ozdemir et al. [10] also showed that such differentiation in MIDs is a function of isolator characteristics (characteristic strength to weight ratio and isolation period). Thus, there is a need to investigate the adequacy of the existing approximate methods to predict the response of LRBs when the effect of lead core heating is taken into account.

Research Objectives

The objectives of the present study are (i) to assess the accuracy of the existing formulations for simplified analysis of SISs; (ii) to determine the relations between the dissipated energies of LRBs represented by both non-deteriorating ($E_{non\text{-}deteriorating}$) and deteriorating ($E_{deteriorating}$) hysteretic representations; (iii) to improve the accuracy of existing simplified methods by translating these energy relations into a proposed equation that will be used in preliminary design of, specifically, structures isolated with LRBs where lead core heating effect is of concern.

To achieve the purpose of the study presented herein, several NTHA and simplified analyses are conducted. In NTHA, the temperature dependent behavior of LRBs is employed by means of a deteriorating force-deformation relation that is capable of representing the instantaneous reduction in the strength of bearing as a function of rise in lead core temperature under cyclic motion. The analyses are conducted by structural analysis program OpenSees [12]. During the analyses, significance of both isolation period and characteristic strength of isolators are investigated.

Analyzed Structural Model

In order to achieve the purpose of the study, several parametric analyses with various isolation systems were performed with a 3-story seismic isolated steel frame structure. The structure was adopted from the hypothetical emergency operation center designed by Dr. Charles A. Kircher for National Earthquake Hazards Reduction Program (NEHRP) [13]. The considered building has a total height of 9m with 3m story height at each floor and is symmetric in plan. The structure has plan dimensions of 36mx54m and there are four and six bays in short and long directions, respectively. All bays are identical with a span length of 9m. Total weight of the superstructure is 73000 kN. Weight of the floor at roof level is taken as 75% of the weight of the other floors and weight at the isolation level is assumed to be equal to the first and second floor weights. The structure is regular in elevation and symmetric with respect to two main orthogonal axes both in mass and stiffness. Analytical model of the investigated seismic base isolated structure is illustrated in Fig. 1.

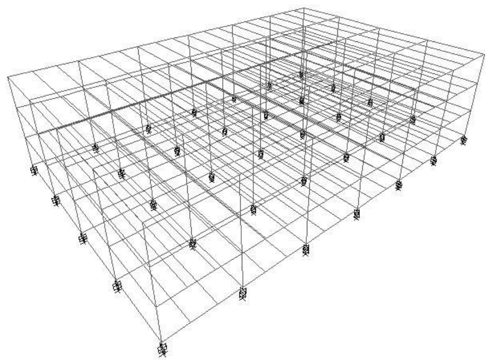

Figure 1. Analytical model of the seismic base isolated structure.

Selected Ground Motions

In this study, considered ground motion set is composed of 40 near-field records and taken from the study of Gunay and Sucuoglu [14]. All of the considered ground motion records have a closest distance (*R*) to the fault rupture less than 20 km which is assumed to be near-field zone [15]. The corresponding moment magnitudes (M_w) of the selected records are in between 6.2 and 7.5. The average shear wave velocities of the selected records at the uppermost 30 m (V_{s30}) soil profile varies in between 180 m/sec to 360 m/sec.

Scaling of Selected Ground Motions

The present study follows a scaling process compatible with the specifications of American Society of Civil Engineering (ASCE) [16]. It assures that the selected ground motions should be scaled such that for the period range under interest, the average of the 5% damped spectral ordinates from all ground motions does not fall below 1.3 times the corresponding 5% damped target spectrum by more than 10%. ASCE [16] also declares that if seven or more ground motion records are used in the analyses, the average value of the response parameter of interest can be used. The above statements are achieved by following a scaling method composed of two complimentary steps with ground motions more than seven. In the first step, 5% damped acceleration spectrum of each ground motion record becomes compatible with the target spectrum. The second step of the scaling was performed to assure the requirements of ASCE [16]. For detailed information about the procedure, see Ozdemir and Constantinou [5]. Fig. 2 shows the scaled average spectrum of those ground motions together with the 5% damped design spectra. The minimum, maximum, and average of scale factors used in this study are, respectively, 0.8, 4.3, and 2.6.

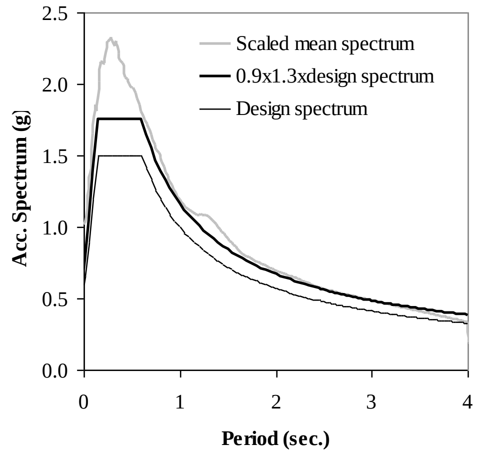

Figure 2. Design spectrum and mean spectrum of ground motions after scaling.

Deteriorating Hysteretic Behavior of LRBs

The nonlinear force-deformation relation of LRBs is generally represented by a generic non-deteriorating bilinear hysteretic model. However, experimental studies showed that hysteretic behavior of LRBs deteriorates under cyclic motion [17]. Basis of such variation in strength of isolators has been identified so far by the effects of loading history, aging, contamination and heating. On the other hand, Kalpakidis and Constantinou [7,8] revealed that most of the reduction in strength of LRBs occurs due to heating of lead core. Authors also proposed a mathematical model that is capable of simulating the deterioration in strength of LRBs as a function of the lead core temperature. Their model considers the instantaneous temperature rise in the lead core due to cyclic motion of LRBs and allows calculating the reduction in strength of isolator via reducing the initial yield stress of the lead, instantly as described in Eq. (1).

$$\sigma_{YL}(T_L) = \sigma_{YL0} \cdot \exp(-E_2 \cdot T_L) \tag{1}$$

where, T_L, $\sigma_{YL}(T_L)$ and σ_{YL0} are the instantaneous lead core temperature, instantaneous yield stress of lead core, and initial yield stress of the lead core, respectively. E_2 is a constant that relates the temperature and yield stress.

Once the yield stress of lead is defined as a function of instantaneous lead core temperature, the force carried by LRB, F_b, can be calculated as described in Eq. (2) and (3).

$$F_b = k_d D_y + \sigma_{YL}(T_L) A_L Z \tag{2}$$

$$D_y \dot{Z} = \left(A - |Z|^2 B\left(1 + \mathrm{sgn}\left(\dot{D}Z\right)\right)\right)\dot{D} \tag{3}$$

where A_L is the cross-sectional area of the lead core, k_d and D_y are the post-yield stiffness and yield displacement of the bilinear force-deformation relation, respectively (see Fig. 3a). In Eq. (2), Z satisfies the first-order differential equation given in Eq. (3) where A and B are dimensionless quantities that control the shape and size of the hysteresis loops of the bearing and are equal to 1.0 and 0.5, respectively.

Design of Analyzed Bearings

In order to assess the effect of isolator characteristics on the accuracy of simplified method of analysis to estimate MIDs, 16 LRBs were designed. These bearings have various isolation periods and characteristic strength to weight ratios to be able to perform parametric analyses. Each of these isolators was designed following an iterative procedure, which will be discussed in the following section in detail, that fulfills the requirements for stability and strength of bearings. Analyzed LRBs were designed in accordance with the design spectrum given in Fig. 2. Isolators were designed with post-yield isolation periods, T_d, of 2.25s, 2.50s, 2.75s, 3.00s. The characteristic strength to weight ratios of the designed LRBs are 0.075, 0.090, 0.105, 0.120. The properties of the designed LRBs are given elsewhere [18].

Equivalent Linearization of Bilinear Hysteretic Behavior

The isolation systems considered in this study are composed of LRBs and are typically represented by a generic bilinear hysteretic curve (Fig. 3.a). Such a representation is steady-state and does not consider any deterioration in strength of isolator under cyclic motion. In Fig. 3.a, k_e is the initial elastic stiffness; k_d is the post yield stiffness; Q is the characteristic strength of the bearing; F_y and U_y are yield strength and yield displacement of the bearing, respectively; F and U are maximum force acting on the bearing and maximum isolator displacement, respectively; k_{eff} is the effective stiffness.

Equivalent linearization is an attempt to simplify the nonlinear behavior (Fig. 3.a) of systems by assuming a representative linear system so that the maximum displacement response of both equivalent linear and nonlinear systems matches. This is achieved by defining effective stiffness, k_{eff}, and equivalent damping ratio, ξ_{eq}. Although there are several methods in literature to define equivalent linear parameters, secant stiffness method is the most commonly used analytically based method. According to secant stiffness method, k_{eff} is defined as the slope of the dashed line that connects the origin to the point of maximum displacement and force as shown in Fig. 3.a and simply calculated by maximum force, F, divided by maximum displacement, U. The corresponding equivalent damping ratio, ξ_{eq}, is determined based on the assumption that the hysteretic energy dissipated at one cycle of isolator motion is equal to the viscous energy dissipation of the equivalent linear system. This implies that the area under the bilinear force-deformation relation given in Fig. 3.a is equal to the area under the curve shown in Fig. 3.b [19]. As a result, ξ_{eq} is derived as:

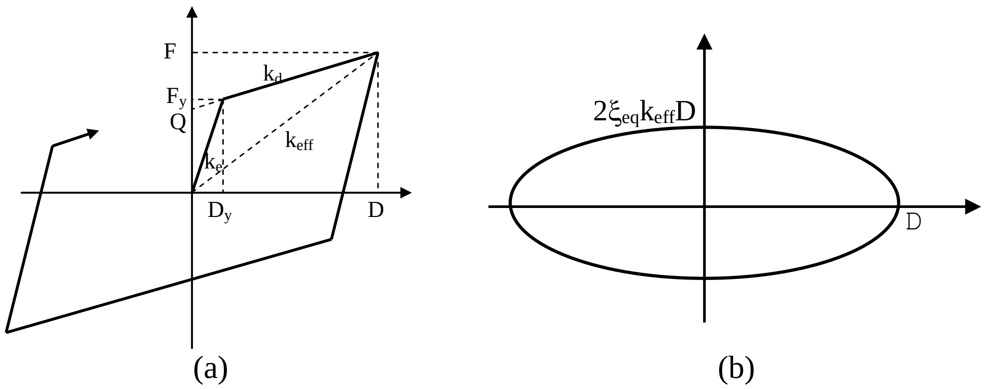

Figure 3. (a) Idealized non-deteriorating hysteresis of a typical LRB (b) Hysteresis of a viscous damper.

$$\xi_{eq} = \frac{4Q(U - U_y)}{2\pi k_{eff} U^2} \quad where \quad k_{eff} = \frac{F}{U} = \frac{Q}{U} + k_d \tag{4}$$

Existing Simplified Methods in the Literature

In the literature, there exists several methodologies, both analytical and empirical, to determine equivalent linear parameters k_{eff} and ξ_{eq} to be used in simplified method of analysis. The secant stiffness method, which is discussed in the previous section, is the most commonly used method to idealize the nonlinear hysteretic behavior of isolators by an equivalent linear system. However, due to the assumptions involved in secant stiffness method, there have been numerous researches that investigate its accuracy to predict the maximum response quantities of the corresponding nonlinear isolation system. In the following subsections, previously proposed formulations developed to increase the accuracy of the simplified method of analysis are discussed briefly.

Hwang and Sheng, 1993 (termed as Hwang 1993)

In their study, Hwang and Sheng [20] focused on the linearized system parameters, effective stiffness and equivalent damping ratio, especially, for LRBs. Authors proposed a set of formulation for both k_{eff} and ξ_{eq}. The empirical formulations proposed by Hwang and Sheng [20] are given in Eq. (5) and (6), where k_e is the initial elastic stiffness, k_{eff} is the effective stiffness, ξ_{eq} is the equivalent damping ratio, and μ is the ductility ratio defined as MID, D, over yield deformation, D_y.

$$k_{eff} = \left[1 + \ln\left[1 + 0.13(\mu - 1)^{1.137}\right]\right]^{-2} \cdot k_e \tag{5}$$

$$\xi_{eq} = 0.0587(\mu - 1)^{0.371} \tag{6}$$

Hwang et al., 1995 (termed as Hwang 1995)

Hwang et al. [21] proposed a similar formulation to increase the accuracy of the simplified method of analysis to predict the MID of LRB isolated bridges. The empirical formulations proposed by Hwang et al. [21] for effective stiffness and equivalent damping ratio are given through Eq. (7) and (8), respectively. In these equations, α is the ratio between post-yield stiffness, k_d, and initial stiffness, k_e.

$$k_{eff} = \frac{1+\alpha(\mu-1)}{\mu}\left[1-0.737\frac{\mu-1}{\mu^2}\right]^{-2}\cdot k_e \tag{7}$$

$$\xi_{eq} = \frac{2(1-\alpha)\left(1-\frac{1}{\mu}\right)}{\pi[1+\alpha(\mu-1)]}\cdot\frac{\mu^{0.58}}{6-10\alpha} \tag{8}$$

Jara and Casas, 2006 (termed as Jara 2006)

The study conducted by Jara and Casas [3] attempts to predict the MIDs of bridges supported on LRBs by proposing an empirical relation to determine equivalent damping ratio of linearized system. Authors used the secant stiffness method to obtain effective stiffness, k_{eff}, and calculate the equivalent damping ratio, ξ_{eq}, by equating the nonlinear displacement response spectrum for a given earthquake to the linear displacement response spectrum. The expression proposed by Jara and Casas [3] to predict the equivalent damping ratio is:

$$\xi_{eq} = 0.05 + 0.05\ln(\mu) \tag{9}$$

Jara et al., 2012 (termed as Jara 2012)

Jara et al. [6] improved the formulation proposed by Jara and Casas [3] to predict the equivalent damping ratio of linearized systems by incorporating the term, η, which accounts for the effect of the soil deposit where ground motions are recorded. Authors stated that η is 0.065 for earthquakes recorded at firm soils and 0.085 for earthquakes recorded at soft soils.

$$\xi_{eq} = 0.05 + \eta\ln(\mu) \tag{10}$$

Dicleli and Buddaram, 2007 (termed as Dicleli 2007)

Dicleli and Buddaram [4] evaluated the equivalent linear analysis of SDOF systems as a function of both ground motion characteristics (intensity and frequency characteristics) and isolator properties. Results of the study conducted by Dicleli and Buddaram [4] demonstrated that there is a need to incorporate the effective period of the structure into the formulations used to predict equivalent damping ratio. For this purpose, authors proposed an empirical formulation for equivalent damping ratio, ξ_{eq}, as given in Eq. (11), where T_e is the initial period of the bilinear hysteretic force-deformation relation based on initial stiffness, k_e. Authors obtained effective stiffness, k_{eff}, using the secant stiffness method.

$$\xi_{eq} = \frac{4Q(U-U_y)}{2\pi k_{eff}U^2}\sqrt{0.41\left(\frac{T_{eff}}{T_e}-1\right)} \tag{11}$$

ASCE, 2005

To encourage the use of seismic isolation technique, existing codes tend to support the use of simplified method of analysis in design of SIS, at least at the preliminary design stage. Being one of the most commonly used code for seismic isolated structures, ASCE [16] defines the secant stiffness method to calculate both k_{eff} and ξ_{eq} as described in Eq. (4).

Evaluation of Simplified Methods

In this section, the accuracy of the simplified formulations, discussed in previous section, to estimate the MIDs of structures isolated with LRBs is tested when lead core heating is of concern. This is achieved by comparing the MID estimated by simplified method of analysis (D_{SMA}) with the one obtained from nonlinear time history analysis (D_{NTHA}) in which deteriorating force-deformation relations were used to idealize hysteretic behavior of LRBs. Comparisons are done as a function of the isolator characteristics namely, characteristic strength to weight ratio, Q/W, and isolation period, T_d. Results are presented in Fig. 4 in terms of D_{SMA}/D_{NTHA}. The motivation for evaluation of the existing simplified methods to estimate the nonlinear response of LRB isolated structures is to seek the "best" set of equations that may be adopted to consider the lead core heating effects, accordingly. The term "best" is used to define the case where D_{SMA}/D_{NTHA} results are independent of the isolator characteristics, Q/W ratio and T_d, as much as possible.

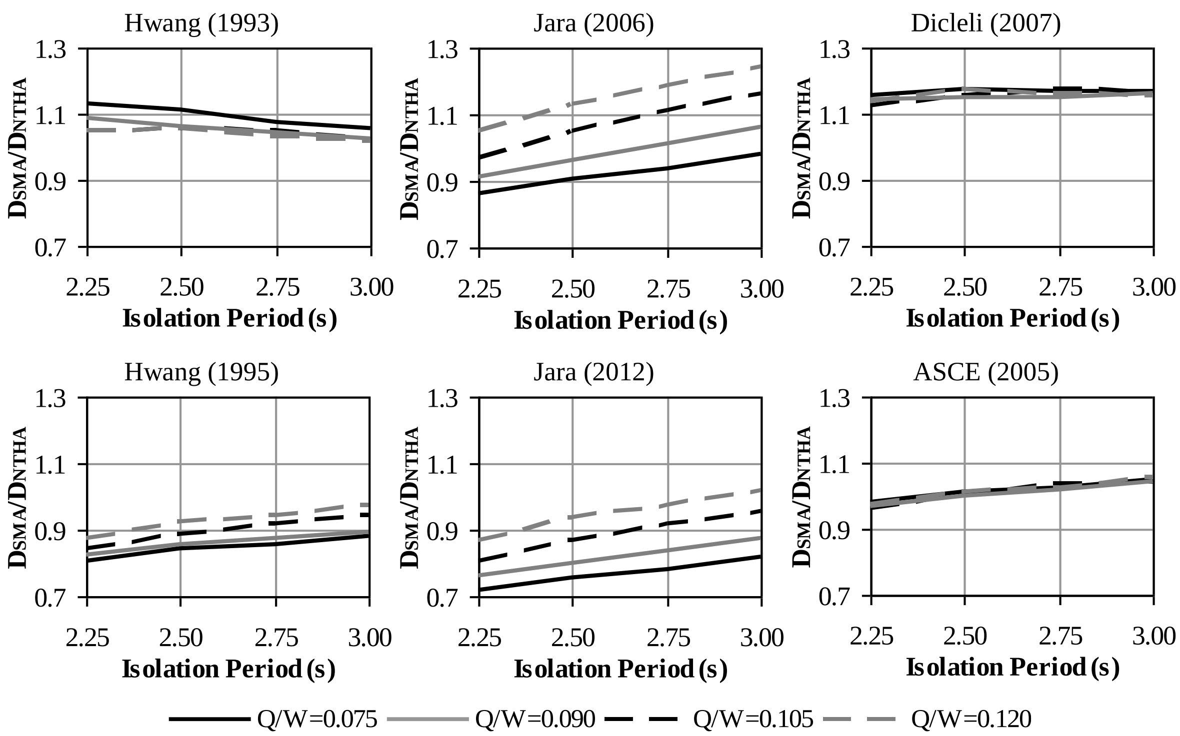

Figure 4. Evaluation of existing formulations to estimate the MIDs.

In Fig. 4, it is clear that D_{SMA}/D_{NTHA} ratios obtained by equations proposed by Hwang (1993), Hwang (1995), Jara (2006), and Jara (2012) depends highly on the isolator characteristics. Although the secant stiffness method employed in ASCE (2005) gives close D_{SMA}/D_{NTHA} values for different Q/W ratios, corresponding D_{SMA}/D_{NTHA} ratios depend on isolation period (D_{SMA}/D_{NTHA} ratios increase with increasing isolation period). The only empirical formulation that has almost the same D_{SMA}/D_{NTHA} ratios for all of the considered isolation characteristics is the one proposed by Dicleli (2007). Thus, the form of the empirical formulation developed by Dicleli (2007) is used as a basis to propose an improved equation to estimate the MIDs.

Relation between $E_{deteriotaing}$ and $E_{non\text{-}deteriorating}$

In this section, relation between the dissipated energies of cases, where both deteriorating and non-deteriorating force-deformation relations of LRBs are employed, is obtained. For this purpose, several NTHA were performed with considered LRB isolated structures. Variations in dissipated energies of deteriorating and non-deteriorating cases are obtained by means of energy ratios $E_{deteriorating}/E_{non\text{-}deteriorating}$ as a function of both Q/W and T_d. Here, both $E_{deteriorating}$ and $E_{non\text{-}deteriorating}$ terms are used to represent the summation of energy dissipations under all excitation cycles of corresponding hysteretic behavior. Corresponding $E_{deteriorating}/E_{non\text{-}deteriorating}$ ratios are tabulated in Table 1. Each data presented in Table 1 represents the averages of computed $E_{deteriorating}/E_{non\text{-}deteriorating}$ ratios. Based on the data given in Table 1, $E_{deteriorating}/E_{non\text{-}deteriorating}$ ratios are incorporated in the proposed equation to estimate the equivalent damping ratio as described in the following section.

Table 1. Average $E_{deteriorating}/E_{non\text{-}deteriorating}$ ratios.

Q/W ratio	**Isolation Period, T_d (sec)**			
	2.25	**2.50**	**2.75**	**3.00**
0.075	1.02	1.03	1.03	1.04
0.090	1.03	1.03	1.03	1.04
0.105	1.02	1.03	1.02	1.03
0.120	1.02	1.02	1.03	1.03

Proposed Empirical Formulation for ξ_{eq}

In light of the performance of existing formulations to predict MIDs, there is a need to improve the accuracy of formulations employed by existing simplified methods to estimate the maximum nonlinear response of LRB isolated structures. In previous sections, the form of the formulation that should be considered in the calculations of equivalent linear systems is determined. The proposed form of formulation to estimate the equivalent damping ratio is as follows:

$$\xi_{eq} = E_r \cdot \frac{4Q\left(U - U_y\right)}{2\pi k_{eff} U^2} \sqrt{\varphi \cdot \left(\frac{T_{eff}}{T_e} - 1\right)} \tag{12}$$

In Eq. (12), the change in energy dissipation capacity of LRBs due to deterioration in isolator strength as a function of lead core heating is incorporated by considering the term E_r. In Eq. (12), E_r stands for the $E_{deteriorating}/E_{non\text{-}deteriorating}$ ratio and equals to 1.03. This value is obtained by taking the average of $E_{deteriorating}/E_{non\text{-}deteriorating}$ ratios presented in Table 1. The term φ in En. (12) is calculated by means of a regression analysis that seeks the minimum least square fit and found to be 0.7.

To display the success of the proposed equation for equivalent damping ratio, ξ_{eq}, to estimate MIDs of LRB isolated structures under investigation, Fig. 5 is depicted. In this figure, MIDs estimated by simplified method of analyses ($D_{proposed}$), where Eq. (12) is used to calculate ξ_{eq}, versus MIDs obtained from NTHA (D_{NTHA}) are depicted as a function of both *Q*/*W* ratio and T_d. The black solid line shown in Fig. 5 represents the cases where $D_{proposed}$ and D_{NTHA} are equal to each other.

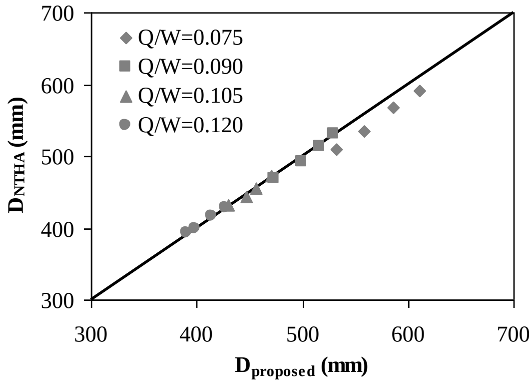

Figure 5. Comparison of $D_{proposed}$ and D_{NTHA}.

Fig. 5 demonstrates that the proposed formulations are highly effective in estimation of MIDs of LRB isolated structures obtained from NTHA. Proposed formulation results in almost the same MIDs with those of NTHA with a slight overestimation which can be neglected. The amount of overestimation in MIDs is less than 5% with the exception of lowest *Q*/*W* ratio. However, it is to be noted that 0.075 is an unrealistically low value for *Q*/*W* ratio in design of LRBs for the corresponding design spectra employed. Fig. 5 also reveals that the accuracy of the estimations for MIDs performed by the proposed formulation is not sensitive to any change in isolation period, T_d, and *Q*/*W* ratio.

Conclusions

In this study, the accuracy of existing simplified formulations to estimate the nonlinear response of seismic isolated structures, isolated with LRBs, in terms of maximum isolator displacements is investigated by incorporating the effect of lead core heating. Results of the present study are used to improve the accuracy of existing formulations to calculate the equivalent damping ratio. For this purpose, a parametric research was conducted with a 3-story representative structure isolated with LRBs. Response of the considered seismic isolated structure subjected to selected near-field ground motions is studied as a function of isolator characteristics namely, isolation period and characteristic strength to weight ratio. Proposed equation to predict the equivalent damping ratio considers the differentiation in energy dissipation capacity of an LRB due to deterioration in its strength as a result of rise in the lead core temperature. It is revealed that simplified analyses in which the proposed equation is used to calculate equivalent damping ratio, yields highly accurate estimates for maximum isolator displacements regardless of the isolator characteristics.

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