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# DEFLECTION AND VIBRATION CRITERIA FOR CONCRETE BRIDGES

E.-S. Hwang<sup>1</sup> and X.H. Le<sup>2</sup>

## ABSTRACT

Deflection and vibration play an important role on the serviceability of bridges. Some codes specify deflection or span-depth ratio limit and others specify vibration limit using natural frequency or acceleration. Bridge vibration concerns are largely based upon human perception. Human perception of vibration depends upon a combination of maximum deflection, maximum acceleration and frequency of response. The paper presents literature review of deflection and vibration criteria, as well as human response to vibration in order to understanding how these limits affect bridge design and user comfort. Actual bridges that present typical types of concrete bridges are selected for experimental measurement and theoretical study. The design documents of the bridges are collected for modeling process. The dynamic deflection and acceleration of these bridges are recorded when the test truck crossing bridges at various speeds as normal traffic. Collected data from field measurement are analyzed in order to determine static deflection and frequency. These values are compared to analytical predictions. Theoretical and experimental results are then compared with design limits as well as other criteria to investigate the applicability of design limit.

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<sup>1</sup>Professor, Dept. of Civil Engineering, Kyung Hee University, Yongin, Korea

<sup>2</sup>Graduate Student Researcher, Dept. of Civil Engineering, Kyung Hee University, Yongin, Korea

# Deflection and vibration criteria for concrete bridges

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Deflection and vibration play an important role on the serviceability of bridges. Some codes specify deflection or span-depth ratio limit and others specify vibration limit using natural frequency or acceleration. Bridge vibration concerns are largely based upon human perception. Human perception of vibration depends upon a combination of maximum deflection, maximum acceleration and frequency of response. The paper presents literature review of deflection and vibration criteria, as well as human response to vibration in order to understanding how these limits affect bridge design and user comfort. Actual bridges that present typical types of concrete bridges are selected for experimental measurement and theoretical study. The design documents of the bridges are collected for modeling process. The dynamic deflection and acceleration of these bridges are recorded when the test truck crossing bridges at various speeds as normal traffic. Collected data from field measurement are analyzed in order to determine static deflection and frequency. These values are compared to analytical predictions. Theoretical and experimental results are then compared with design limits as well as other criteria to investigate the applicability of design limit.

## Introduction

The goal of the bridge engineer is to design bridges which are durable, serviceable and safe. One of the important concerns in serviceability is the comfort of those crossing the bridges. Although human perceptions are subjected to the vibrations, there is seldom direct provision in design codes to ensure the user comfort. The current AASHTO LRFD Bridge Design Code [1] specifications specify span-to-depth ratio and static deflection limits in the hope that these limits will control vibrations. However, the human is primarily sensitive to accelerations rather than displacement [2].

The objective of the paper is to compare and analyze the existing design criteria on deflection and/or vibration limit in various codes and documents and investigate the applicability on typical bridge types in Korea through theoretical and experimental study. This study focuses on literature review of deflection and vibration, human response to vibration in order to understanding how these limits ensure the comfort of bridge users.

Actual bridges that represent typical types of bridge are selected for experimental measurement and theoretical study. The design documents of the bridges are collected for modeling process. The dynamic deflection and acceleration of these bridges are recorded when the test truck or normal traffic cross bridges at various speeds. The collected data from field measurements are analyzed in order to determine static deflection and frequency. These values are compared to analytical predictions. Theoretical and experimental results are then compared with design limit to investigate the applicability of design limit.

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<sup>1</sup>Professor, Dept. of Civil Engineering, Kyung Hee University, Yongin, Korea

<sup>2</sup>Graduate Student Researcher, Dept. of Civil Engineering, Kyung Hee University, Yongin, Korea

## Literature review

### 1. History of deflection limit

The current AASHTO LRFD Bridge Design Code specify the bridge deflection limits at  $L/800$  for vehicular bridges and  $L/1000$  for pedestrian bridges as an optional criteria. In efforts of finding relevant literature, assemble and review from foreign sources, the preliminary literature indicated that the original deflection limits may be traced back to 1871 with the specifications established by the Phoenix Bridge Company (PBC).

In 1905, American Railway Engineering Association [3] specification limits the span-to-depth ratio  $L/D$  as the methods control indirectly the maximum live-load deflection.

In the early 1930's, the conclusion of the Bureau of Public Roads indicated that the exceeded limit of  $L/800$  will cause unacceptable vibration on structures and the unacceptable vibration is determined by subjective human response. This study specified  $L/800$  as the deflection limit.

In the report of American Society of Civil Engineers [4] on bridge deflection limits, span-to-depth ratio limits and deflection limits for highway bridges are specified as the method controlling vibration. The report also showed no relationship between structural damage and live-load deflection.

### 2. Human response to vibration

In the field of transportation, the comfort of automobiles, vehicles is a major concern, so the engineers must consider vibrations of bridges. Human reactions to vibrations are both physiological and psychological.

In 1902, a measurements on human sensitivity to vibration were carried out by Mallock [5]. By investigating complaints of unpleasant vibration caused by passing traffic, he concluded that acceleration is the cause of discomfort. He also found that amplitudes seldom exceeded 0.001 inch and that frequencies ranged from 10 to 15Hz. After that, numerous experiments in the field of human sensitivity to vibration have been carried out. In 1931, Reiher and Meister [6] produced 6 tolerance ranges based on reactions of 25 adult subjects between the ages of 20 to 37 years. The tolerance ranges are classified as (1) not perceptible, (2) slightly perceptible, (3) distinctly perceptible, (4) strongly perceptible or annoying, (5) unpleasant or disturbing, and (6) very disturbing or injurious is shown in Figure 1

After Reiher and Meister, Parmelee and Wiss [7] investigated the effect of damping upon human sensitivity to vibration and found sensitivity to be proportional to the product of maximum displacement and frequency.

In 1948, Goldman [8] reviewed the problem and produced from several different sources, including Reiher and Meister, a set of revised averaged curves corresponding to three tolerance levels classified as perceptible, unpleasant, and intolerable.

In 1984 [9], Janeway's limits recommended that displacement amplitude ( $a$ ) and frequency of vibration ( $f$ ) for various ranges of frequency, as shown in Table 1 and Figure 2.

Wright and Walker [10], based on human responses to harmonic vertical vibrations is shown in Table 2, proposed a vibration related static deflection limit. The limit is a computed transient peak acceleration of a bridge,  $\alpha$ , which should not exceed  $100\text{in./s}^2$  ( $2.54\text{ m/s}^2$ ), where the static deflection,  $\delta$ , is linked to  $\alpha$  as follows:

$$\delta_s = 0.05 \times L \times \frac{\alpha}{(\text{speed} + 0.3 \times f_s \times L) \times f_s} \quad (1)$$

where L span length; speed: vehicle speed and  $f_s$  natural frequency of a simple span bridge.

$$f_s = \frac{\pi}{2L^2} \sqrt{\frac{E_b I_b}{m}} \quad (2)$$

where  $E_b I_b$  flexural rigidity of a girder section; m: unit mass of a girder section. It should be noted that  $\delta_s$  needs to be calculated for a live load with a girder distribution factor of 0.7.

Table 1. Janeway's limits

Janeway's limits recommendation	Frequency (Hz)
$af^3 = 2$	1 - 6
$af^2 = 1/3$	6 - 20
$af = 1/60$	> 20

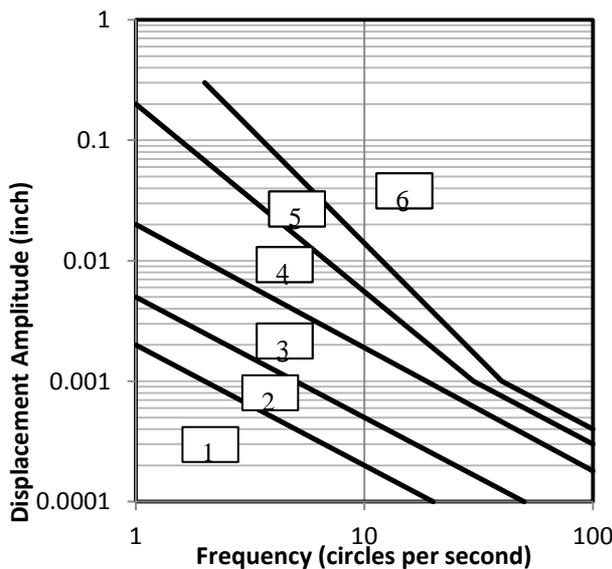


Figure 1. Human levels by Reiher and Meister

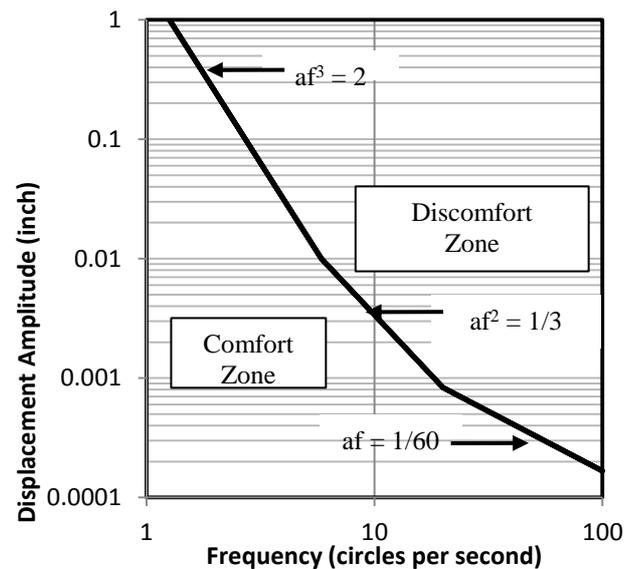


Figure 2. Vibration limits by Janeway

### 3. Live load deflection criteria

#### 3.1. Canadian Standards and Ontario Highway Bridge Code

Both the Canadian Standard and the Ontario Highway Bridge Code [11] use a relationship between natural frequency and maximum static deflection to evaluate the acceptability of a bridge design for the anticipated degree of pedestrian use. The relationship between frequency and deflection in Canadian Standard is shown in Figure 3.

This relationship was developed from extensive field data collection and analytical models conducted by Wright and Green. For highway bridges, acceleration limits were converted to equivalent static deflection limits to simplify the design process. For pedestrian

traffic, the deflection limit applies at the center of the sidewalk or at the inside face of the barrier wall or railing for bridges with no sidewalk.

Table 2. Peak Acceleration for Human Response to Harmonic Vertical Vibration [10].

<b>Human Response</b>	<b>Transient peak acceleration (g)</b>	<b>Sustained Peak Acceleration (g)</b>
Imperceptible	0.013	0.001
Perceptible to Some	0.026	0.003
Perceptible to Most	0.052	0.005
Perceptible	0.129	0.013
Unpleasant to Few	0.259	0.026
Unpleasant to Some	0.518	0.052
Unpleasant to Most	1.295	0.129
Intolerable to Some	2.589	0.259
Intolerable to Most	5.178	0.518

### ***3.2. Eurocodes***

There is no deflection limit criteria for road bridges. Vibration limits of pedestrian comfort for footbridge are shown as following:

A verification of the comfort criteria should be performed if the fundamental frequency of the deck is less than:

- 5Hz for vertical vibrations,
- 2.5Hz for horizontal (lateral) and torsional vibrations.

In this code, there is a specification of deflection limit for railways bridges: the maximum total vertical deflection measured along any track due to rail traffic actions should not exceed  $L/600$ , where  $L$  is span length.

### ***3.3. British Standards***

In British Standards, vibration serviceability requirements of foot and cycle track bridges are specified. When the fundamental natural frequency of vibration exceeds 5Hz for the unloaded bridge in the vertical direction and 1.5Hz for the loaded bridge in the horizontal direction, the vibration serviceability requirement is deemed to be satisfied.

### ***3.4. New Zealand***

In New Zealand, the 1994 Transit NZ Bridge Manual limits the maximum vertical velocity to 0.055 m/s (2.2 in/s). Older versions of this Bridge Manual also employed limits on  $L/D$  and deflection, but these are no longer used in design.

### ***3.5. Australian Codes***

In Australian Codes [12], the relationship between first mode flexural frequency (Hz) and

static deflection (mm) is shown in Figure 4.

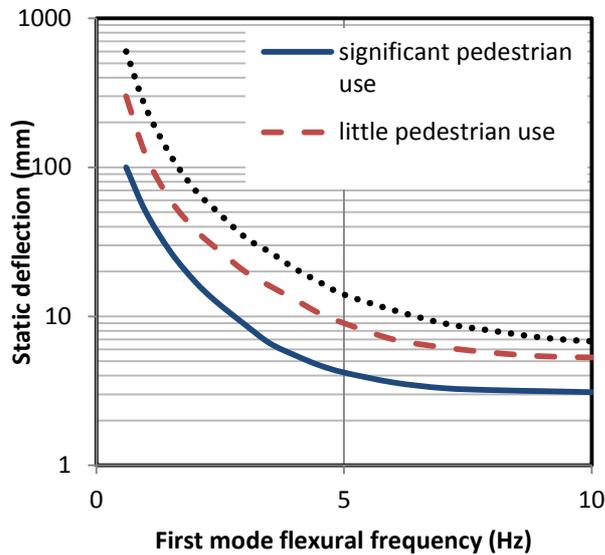


Figure 3. First Flexural Frequency versus Static Deflection in Canadian Code [11].

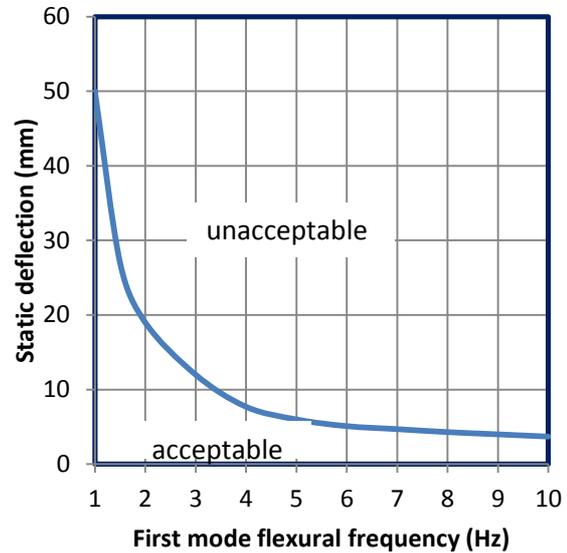


Figure 4. First Flexural Frequency vs Static Deflection in Australian Code [12].

#### 4. Summary:

In AASHTO LRFD, the specification of limiting span-to-depth ratio and limiting deflection to  $L/800$  for most design situations and  $L/1000$  for urban areas where  $L$  is the span length, have been employed for many years since 1871 and no major change have been made for highway bridges since 1936. American Society of Civil Engineers (ASCE) committee in 1958 reviewed historical of deflection limit and investigated the effect of deflection to structural damage, they concluded that no relationship between excessive live-load deflection and structural damage and the limiting of deflection seems to be a method controlling vibration. Bridge vibration concerns are based upon human perception, human perception of vibration depends on velocity of vehicles, deflection, acceleration, frequency. Therefore, in order to control human perception, many criteria on deflection, acceleration, velocity, frequency have been issued. Oriard (1972) specified limit of velocity, Wright and Walker (1971) produced many level of acceleration, Reiher and Meister, Janeway developed the relationship between natural frequency and maximum static deflection to evaluate the human response. Those limits are also adopted in Canadian and Australian Codes. Many references show that the relationship between displacement and frequency as criteria for vibration serviceability.

The relationships between deflection and frequency specified by Reiher and Meister, Janeway, Canadian code, Australian Code are plotted on the same chart as shown in Figure 5.

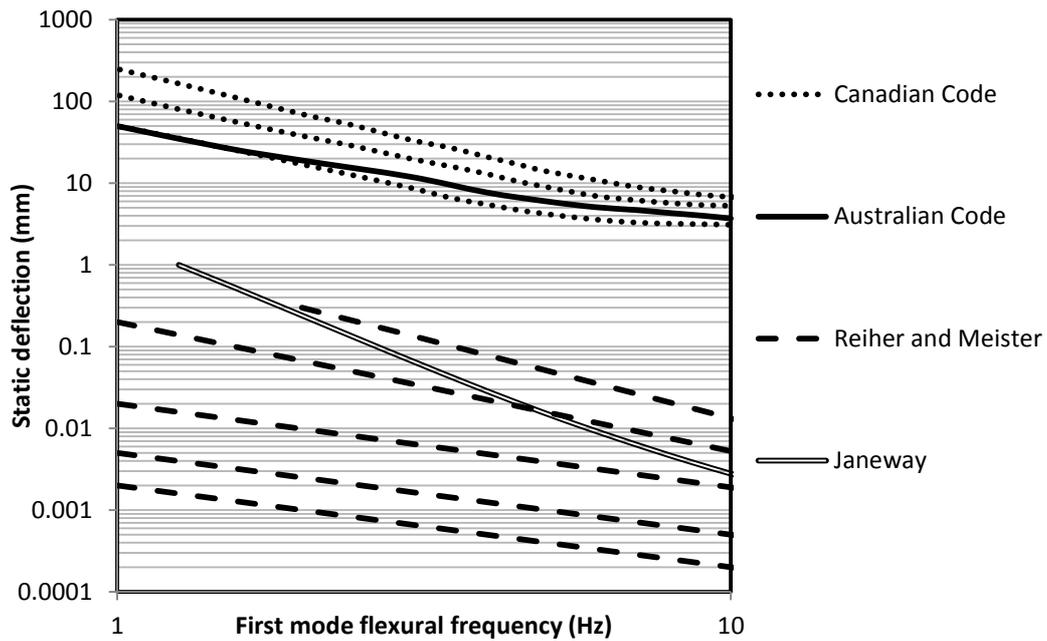


Figure 5. Deflection versus frequency in various codes.

### Experimental program

The bridges included in this study are given in Table 4. Seven bridges that represent typical bridge types were selected for the field measurement. Especially, two of those use Ultra High Performance Concrete (Bridge No.7).

Table 4. Seven Bridges Selected for the Testing Program.

No.	Name	Type of construction	Span length (m)
1	Keumdang	PSC Box Girder	38
2	Keumdang	PSC Beam	30
3	Gupo	Rahmen Bridge	13
4	Simiyug	PSC Beam	30
5	Songjeon-yug	PSC Beam	30
6	Byeogjeyug	PSC Box Girder	45
7	Andong 1	Simple PSC Beam	11

The bridges were instrumented with accelerometers and Linear Voltage Displacement Transducer (LVDT). The LVDT are installed at the mid of span of exterior girder where give the maximum deflection in order to record dynamic deflection when vehicles cross. Accelerometers are also placed at the same longitudinal location of LVDT and near edge of bridge and, where pedestrians use in order to record acceleration of bridges.

The test truck ran to cross the bridges with various speed as normal traffic. Dynamic deflection and acceleration of bridge due to test truck are collected as shown in Figure 6.

## Analysis results

### 1. Experimental analysis

Collected data from experimental testing including dynamic deflection and acceleration are used to determine static deflection and natural frequency. Static deflection is determined as live-load deflection when the test truck running at very slow speed (10kmph).

First natural frequency is also determined by using Fast Fourier Transform (FFT) algorithm from acceleration data as illustrated in Figure 7.

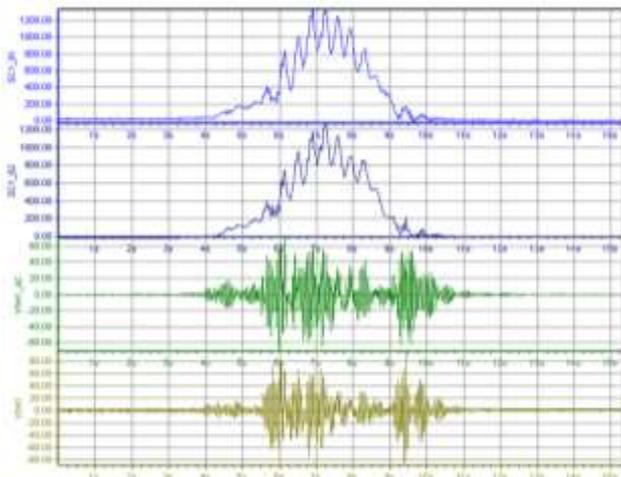


Figure 6. Dynamic deflection and acceleration collected from experimental measurement

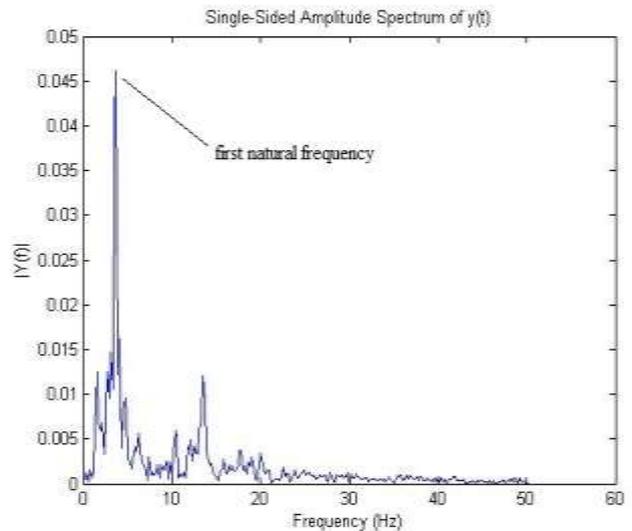


Figure 7. Determination of first natural frequency by FFT algorithm

### 2. Comparisons to various comfort criteria

The Wright and Walker recommendations determine an effective allowable peak acceleration based on the fundamental natural. The value of this peak acceleration is then compared against tabulated limits shown in Table 1. If the peak acceleration exceeds  $100 \text{ in./s}^2$  the member is to be redesigned. The comparisons with Wright and Walker Criteria are shown in Table 5.

### 3. Comparison with AASHTO Standard Specifications

The selected bridges are modeled on Midas Program for theoretical analysis. Deflection due to design truck (DB24) is also calculated in order to compare with AASHTO deflection limit  $L/800$ . All deflection values are shown in Table 6.

### 4. Comparison with other design codes

First natural frequency calculated from FFT algorithm and deflection due to design truck (DB24) after being multiplied by respond ratio are used to plot on Reiher and Meister criteria, Janeway criteria, Ontario Highway Bridge Design Code and Australia Code, then they are shown in Figure 8, 9, 10 and 11, respectively.

Figure 9 and 10 show almost selected bridges are failing to meet the limits of combination of displacement and frequency on Reiher, Meister and Janeway criteria, while they are satisfied if compared to AASHTO and Australian code except bridge No.7 which uses high strength material.

Table 5. Comparisons with Wright and Walker Criteria.

<b>Bridge #</b>	<b>Name</b>	<b>Frequency Hz</b>	<b>Acceleration (80km/h) in/s<sup>2</sup></b>	<b>Wright and Walker Human response</b>
1	Keumdang	2.9	4.57	Imperceptible
2	Keumdang	2.55	3.51	Imperceptible
3	Gupo	13	3.27	Imperceptible
4	Simiyug	3.7	6.07	Perceptible to Some
5	Songjeon-yug	3.7	7.17	Perceptible to Some
6	Byeogjeyug	1.5	1.10	Imperceptible
7	Andong 1	10.5	69.74	Unpleasant to Few

Table 6. Comparison with deflection limit.

<b>Bridge #</b>	<b>Name</b>	<b>L (m)</b>	<b>Measured Deflection (mm)</b>	<b>Calculated deflection (mm)</b>			<b>L/800 (mm)</b>
				<b>Test truck</b>	<b>DB24</b>	<b>DB24 after adjusted</b>	
1	Keumdang	38	1.01	1.07	2.95	2.78	47.5
2	Keumdang	30	1.47	1.6	3.47	3.19	37.5
3	Gupo	13	0.19	0.21	0.57	0.52	16.25
4	Simiyug	30	0.82	1.2	3.18	2.17	37.5
5	Songjeon-yug	30	0.80	1.2	3.18	2.12	37.5
6	Byeogjeyug	45	0.67	1.2	4.20	2.35	56.25
7	Andong 1	11	2.92	3.82	4.33	3.31	13.75

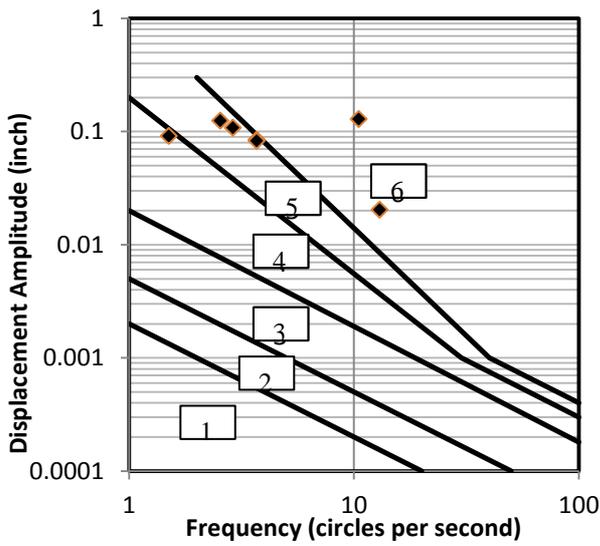


Figure 8. Human levels by Reiher and Meister

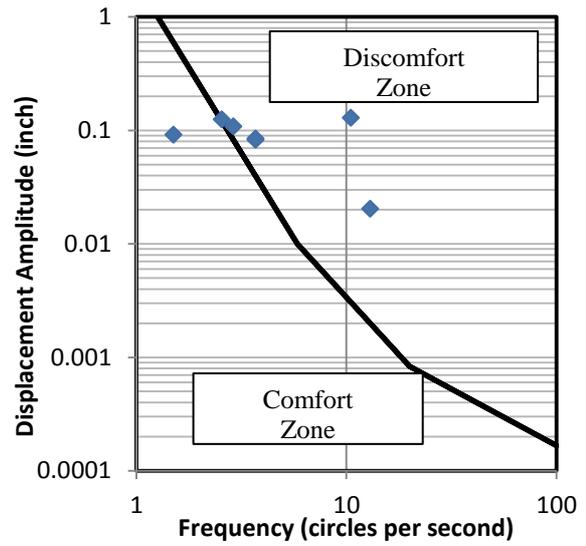


Figure 9. Vibration limits by Janeway

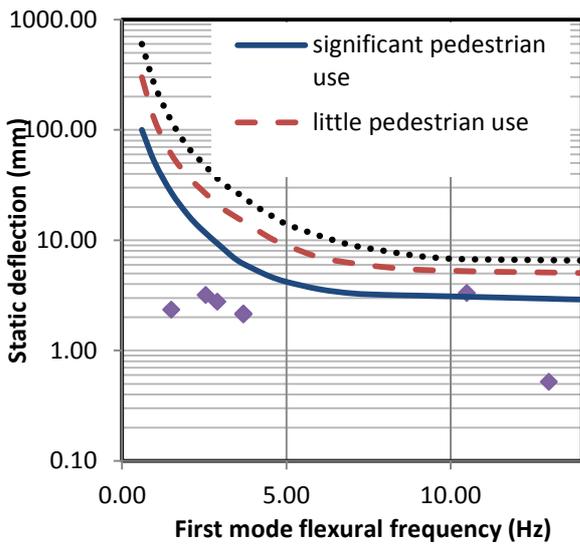


Figure 10. Static deflection and frequency comparing to Canadian code

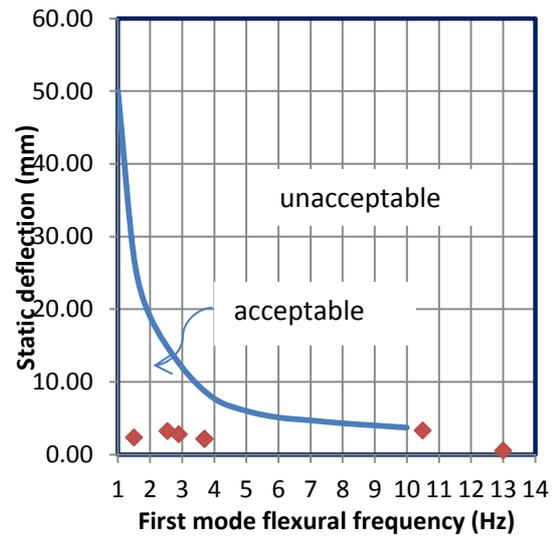


Figure 11. Static deflection and frequency comparing to Australian code

### Conclusions

This paper presents results from evaluation of the deflection and vibration criteria from criteria and design codes. Based on the analysis results, the following conclusions could be made:

- Relationship between deflection and frequency are commonly used as method to control deflection/vibration and human perception.
- The deflection limits and span-to-depth ratio limits specified in AASHTO code is well above the actual design values and give no effect in design.
- All results from test bridges show that all bridges are very disturbing level by Reiher and

Meister and discomfort zone by Janeway. However, by design code criteria, they satisfied design limit except test bridge No.7 which are high strength material bridge.

- The bridges using high strength material may have possibility to give unacceptable level. Special consideration may be required in using high strength material in bridge design.

### Acknowledgments

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