

# FLEXURAL STRENGTH ASSESSMENT OF CORRODED REINFORCED CONCRETE (CRC) BEAMS

Fares Jnaid<sup>1</sup> and Riyad S. Aboutaha<sup>2</sup>

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This paper studies the effects of corrosion on the ultimate flexural strength of steel reinforced concrete (RC) beams. A Finite Element (FE) model was developed and validated against available experimental data. The FE model was employed to expand the database. Based on the data obtained from the Finite Element Analysis (FEA), a detailed analytical model was developed. The analytical model can predict the ultimate flexural strength of corroded RC beams with different corrosion rates and lengths with decent accuracy. The analytical model was compared to the FE model and the available experimental data and showed good agreement. The analytical model was then employed to investigate the following parameters: corrosion rate, length of corroded zone, span to depth ratio, tension reinforcement ratio, concrete compressive strength, and type of applied load. Moreover, a simplified analytical model was developed to calculate the ultimate flexural strength of corroded reinforced concrete (CRC) beams. The model simplified model showed good agreement with the available experimental data, FE model, and detailed analytical model. It was concluded that for beams subjected to low corrosion rates, the loss of bond between steel reinforcement and the surrounding concrete is the main cause of flexural strength reduction, however, for beams with high corrosion rates, the decrease in ultimate capacity is caused by the decrease in steel's mechanical properties and cross-sectional area.

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## Introduction

One of the major problems that civil engineers face in practice, especially when dealing with steel reinforced concrete members, is the corrosion of reinforcing steel along with concrete deterioration. Corrosion of steel in a reinforced concrete structure leads to cracking and spalling of the concrete cover, reduction in the steel cross-sectional area, loss of concrete integrity, and loss of bond between the steel bars and surrounding concrete. This reduces strength and ductility, which shortens the service life of the structure. Nearly 25% of the bridges in the United States of America are structurally deficient or functionally obsolete (U.S. Department of Transportation, Bureau of Transportation Statistics, 2008). However, due to the lack of research in evaluating the residual strength of steel reinforced beams, there is a need to develop a method to estimate the residual strength of such deteriorated members. This research presents an FEA model, detailed analytical model, and simplified analytical model to calculate the residual capacity of CRC beams.

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## Background

The effects of corrosion on RC beams have been investigated experimentally [1, 2, 3, and 4], numerically [4, 5], and analytically [7, 6].

El Maaddawy et al. [2] performed a test on one control beam and four corroded beams with different corrosion levels. Each beam has a span of 3,000 mm (118 in.), with a cross-section of  $152 \times 254$  mm ( $5.9 \times 10$  in.). The tensile steel reinforcement consists of two No. 15 deformed bars, while the compressive reinforcement consists of two 8 mm (0.3 in.) plain bars. The transverse reinforcement consisted of 8 mm (0.3 in.) stirrups spaced at 80 mm (3.2 in.) in the shear span, and 333 mm (13.1 in.) in the constant moment zone. The 28-day concrete compressive strength averaged 40.5 MPa (5.87 ksi), and the yield and ultimate strength of the tensile steel were 450 and 585 MPa (65.3 and 84.9 ksi) respectively. The yield and ultimate strength of the compressive steel and stirrups were 340 and 500 MPa (49.3 and 72.5 ksi) respectively. While the control beam was not exposed to corrosion, the other four specimens CN-50, CN-110, CN-210, and CN-310 were subjected to 50, 110, 210, and 310 days of corrosion exposure, which resulted in average steel mass losses of approximately 8.9, 14.2, 22.2, and 31.6% respectively.

Du et al. [3] tested 19 reinforced concrete beams, of which 8 specimens were corroded on the tension side. The non-corroded beams were designed to fail in flexural mode under two-point loads. The specimens had dimensions of  $150 \times 200 \times 2100$  mm ( $5.91 \times 7.88 \times 82.67$  in.) and a span of 1800 mm (70.87 in.). One of the beams was over-reinforced, three were balanced reinforced, one was under-reinforced, and three were very under-reinforced. The nominal concrete cover to the longitudinal bars was 20 mm (0.79 in.).

Wang and Chen [4] tested three naturally corroded RC beams with dimensions of  $400 \times 800 \times 6,000$  mm ( $15.75 \times 31.49 \times 236.22$  in.), which were taken from a coastal dock built in 1980. The nominal diameter of longitudinal rebar was 20 mm (0.79 in.) with yielding strength of 335 MPa (48.59 ksi) and ultimate strength of 490 MPa (71.07 ksi). The concrete clear cover was 25 mm (0.98 in.). The average concrete compressive strength was 33.1 MPa (4.8 ksi). The corrosion rate as a percent of mass loss was investigated in different parts of the corroded beams and varied from 0 to 63.3% based on the location of the reinforcement bars. The ultimate load of the three tested beams was reported as 23, 24, and 24 kN (5.17, 5.39, and 5.39 kips). The average strength of three tested beams was 35.4 kN.m (26.11 kip.ft). Moreover, Wang and Chen (2011) performed an FEA study on 7 beams with different corrosion lengths and different corrosion rates.

Horrigmoe and Hansen [5] performed an FEA study using the commercial software ANSYS to investigate the behavior of corroded RC beams with only tensile reinforcement. In order to compute the ultimate capacity of a corroded beam, they implemented the reduction in steel cross-sectional areas due to corrosion and the bond deterioration between corroded steel and surrounding concrete. Their model showed good agreement with experimental data.

Wang and Chen [4] developed a finite element model to estimate the residual bending strength of heavily deteriorated concrete beams. The nonlinear finite element program ABAQUS (2004) was used in this study. The program accounts for element type and mesh, as well as damaged material properties, boundary conditions, and load type. The model was developed to study the effects of reinforcement corrosion, concrete cover spalling,

and midspan vertical crack on the ultimate capacity of concrete beams subjected to corrosion. It was observed that length of exposed steel reinforcement does not affect the strength of concrete beams, but it does affect rigidity. In addition, it was concluded that the decrease in the rebar cross-section is accompanied with a decrease in the strength of reinforced concrete members.

Rodriguez et al. [6] used the reduced cross-section areas to suggest a model based on *Eurocode 2* (EC2) equations. In fact they suggested two methods, one where the concrete cover on all faces is ignored, and the other where the concrete cover is ignored only on the top and bottom of the cross-section. The analytical model was compared to the experimental data carried out by the same researchers. They noted that for beams where bending controlled the failure, the two cross-sections mentioned above demonstrated a decent agreement with the experimental data. However, for beams where shear failure dominated, the cross-section where cover was ignored on all four faces underestimated the ultimate capacity by 30%. Moreover, even when the gross cross-section was taken into account, the results were found to be more comparable to experimental data than when the cross-section was ignored on all four sides.

Azher [7] performed an experimental study on 56 reinforced concrete beams  $150 \times 150 \times 1100$  mm ( $5.91 \times 5.91 \times 43.31$  in.), of which 48 were subjected to corrosion by impressed current. Based on the experimental data, a simplified two step model to estimate the ultimate capacity of corroded beams was presented. The model accounts for corrosion current density, corrosion period, beam cross-section, and material properties. Their model was able to reasonably predict the ultimate capacity of corroded beams. However, the model was based on an empirical equation obtained from the plot of the experimental data.

## **FEA Model**

### **Building the FEA Model**

In order to simulate the behavior of corroded beams, a finite element model was developed, and the results were verified with experimental data by others. After being verified against experimental data, the FEA model was employed to investigate a total of 145 reinforced concrete beams that covered many variables: span to depth ratio, tensile reinforcement ratio, compressive reinforcement ratio, concrete compressive strength, length of corroded zone, and type of applied load.

### ***Element Types***

In order to build the FEA model, three types of elements were used: a 3-D SOLID65 element to model concrete, a 3-D LINK180 element to model steel reinforcement, and a COMBIN14 spring element to model the loss of bond between steel and surrounding. The SOLID65 is defined by eight nodes, each of these nodes have three degrees of freedom; translations in the nodal x, y, and z directions. In addition, this element is capable of cracking under tension and crushing in compression. It also incorporates steel rebars, which makes it ideal for reinforced concrete modeling. The Link180 element is a uniaxial tension-compression defined by two nodes with three degrees of freedom at each node: translations in the nodal x, y, and z directions. This element is capable of carrying tension and compression. The COMBIN14 is a spring element that has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications.

The element has no mass and the spring or the damper capability can be deactivated [8].

### **Material Properties and Real Constants**

The modified Hognestad stress-strain diagram was assumed to define the multilinear isotropic concrete stress-strain curves as required by ANSYS, whereas an elasto-plastic stress-strain relationship was used to define the stress-strain diagram of steel reinforcement as shown in Figure 1. The modulus of elasticity of concrete was 4,750 times the square root of concrete cube strength, uniaxial cracking stress (modulus of rupture) was 8.5% of the compressive strength, and Poisson's ratio was assumed to be 0.2. ANSYS assumes a linear stress-strain relationship for concrete in tension until the uniaxial cracking stress is reached. The modulus of elasticity of steel was 200,000 MPa (29,000 ksi), and Poisson's ratio was 0.3. The steel yield stress varied based on each experiment.

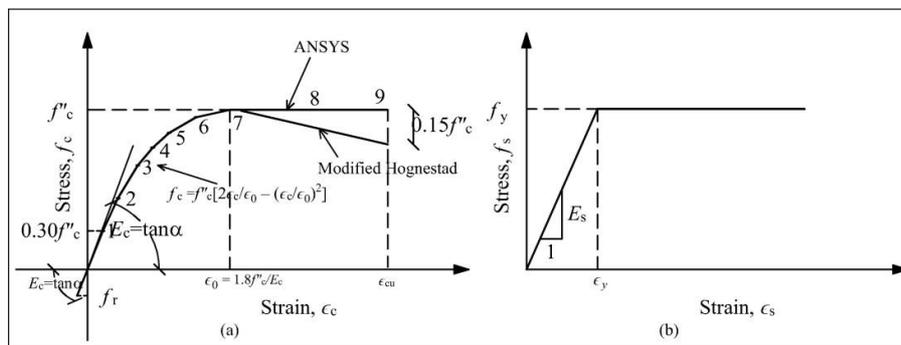


Figure 1. Stress-strain diagrams: (a) concrete: modified Hognestad; (b) steel

### **Validation of the FEA Model**

The FEA model was verified against 29 different experimental beams, of which two were fully bonded, 12 were exposed to different corrosion levels, and the rest were subjected to unbond between steel and surrounding concrete. The experimental data was collected from the following researchers [2, 9, 10, and 11]. While El Maaddawy et al. [2] and Due et al. [11] performed experiments on corroded beam specimens, Cairns and Zhao [9] and Sharaf and Soudki [10] tested beam specimens subjected to unbond between steel bars and the surrounding concrete. Figure 2 shows a comparison between the FEA and the experimental load-deflection curve obtained by El Maaddawy et al. [2] for a beam with a corrosion rate of 8.9% and a corrosion length of 47% of the span, whereas Figure 3 shows a comparison between the FEA and experimental ultimate flexural strengths obtained by the above researchers [2, 9, 10, and 11].

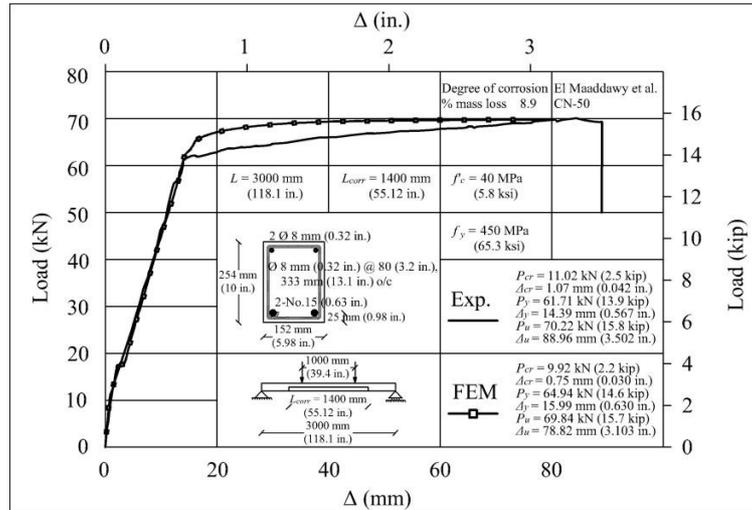


Figure 2. Load-deflection curve of El Maaddawy et al. [2] CN-50 (experimental vs. FEM)

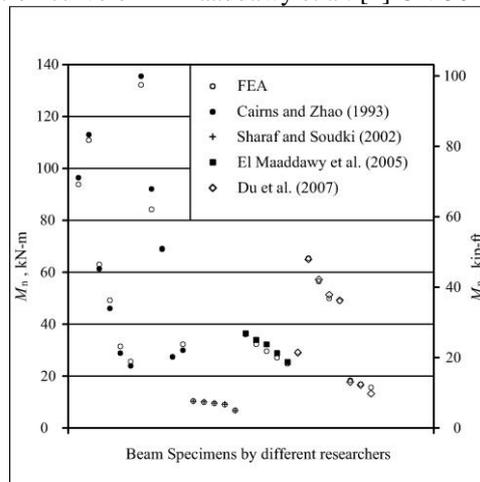


Figure 3. Comparison of FEA with experimental results

## Analytical Model

### Building the Analytical Model

A method that is used in calculating the ultimate stress in the tendons of post-tensioned prestressed concrete beams and is based on the depth of the neutral axis has been adopted in this paper. This is because beams with unbonded reinforcement over the full span are similar to prestressed concrete beams with unbonded tendons. The total elongation of the prestressed tendons can be determined from the distribution of the curvature over the full length of the beam. This elongation, which is equal to the total elongation of the adjacent concrete, is primarily caused by the plastic deformation in an equivalent plastic region of length  $L_o$ , assuming that the deformation within the elastic region and the friction between the prestressing tendons and the adjacent concrete are neglected [12]. Moreover, Pannell [12] suggested that the ratio of the length of the equivalent plastic region  $L_o$  to the depth of the neutral axis at ultimate limit state is constant ( $\Psi$ ). Wang and Liu [13] adopted the value of  $\Psi$  that was suggested by Au and Du (2004) to calculate the ultimate strength of partially corroded beams. However, in this paper, the empirical constant  $\Psi$  is evaluated based on the following parameters: steel reinforcement ratio, span to depth ratio, concrete compressive strength, and loading type.

The FEA model was used to study over 100 cases of beams with unbonded steel bars over the full span. The following parameters were investigated: reinforcement ratio, span to depth ratio, concrete compressive strength, and loading type. After running the FEA model for the cases studied, the values of the ultimate moment and strain in steel were obtained, and based on their values the following equations were suggested for the empirical constant  $\Psi$  [14]:

$$\Psi = 0.9523 \cdot (\rho\%)^2 - \left(0.0864 \frac{L}{d} + 4.3945\right) \cdot \rho\% + 0.6734 \cdot \frac{L}{d} + 4.4783 \quad (1)$$

$$\Psi = 0.9664 \cdot (\rho\%)^2 - \left(0.0877 \frac{L}{d} + 4.4921\right) \cdot \rho\% + 0.6786 \cdot \frac{L}{d} + 3.8643 \quad (2)$$

where  $\Psi$  = length of the equivalent plastic region  $L_o$  to the depth of the neutral axis at ultimate limit state,  $\rho\%$  = tensile reinforcement ratio,  $L$  = span of the beam, and  $d$  = distance from extreme compression fiber to centroid of tensile reinforcement. Note that equation (1) is for beams with one third or uniformly distributed load, whereas equation (2) is for beams with Point load.

The FEA model was employed again to study 27 beams that varied from structurally sound beam to beams with different unbonded lengths. After obtaining the value of  $\Psi$  from the above equations, the strain at ultimate can be obtained. Knowing the strain at ultimate for structurally sound beams and beams with unbonded reinforcement over the full span, the following equation was suggested to calculate the strain at ultimate for beams with unbonded steel reinforcement over a portion of the span [14]:

$$\varepsilon_{sub} = \frac{\left(L^3 - 2 \cdot L^2 \cdot L_{ub} + L \cdot L_{ub}^2 + 2 \cdot L_o \cdot L \cdot L_{ub} - L_o \cdot L_{ub}^2\right)}{L^3} \cdot \varepsilon_{cu} \cdot \frac{d - c_{ub}}{c_{ub}} \quad (3)$$

where  $\varepsilon_{sub}$  = strain of tensile unbonded steel reinforcement,  $L$  = span of the beam,  $L_{ub}$  = length of beam over which reinforcement is unbonded,  $L_o$  = length of the equivalent plastic region,  $d$  = distance from extreme compression fiber to centroid of tensile reinforcement,  $c_{ub}$  = distance from extreme compression fiber to the neutral axis of a cross-section with unbonded reinforcement.

Eqs. 1, 2, and 3 allow for calculating the strain at ultimate of beams with unbonded reinforcement. The equations proposed by Du et al. [15], to estimate the reduction in steel cross-sectional area and strength, are adopted in this research:

$$A_{s(corr)} = (1 - 0.01 \cdot Q_{corr}) \cdot A_s \quad (4)$$

$$f_{y(corr)} = (1 - 0.005 \cdot Q_{corr}) \cdot f_y \quad (5)$$

where  $A_{s(corr)}$  = average cross-sectional area of corroded reinforcement,  $A_s$  = initial cross-sectional area of non-corroded reinforcement,  $f_{y(corr)}$  and  $f_y$  = the yield strengths of corroded and non-corroded reinforcement, respectively, and  $Q_{corr}$  = amount of corrosion of reinforcement (%).

## Validation of the Analytical Model

The analytical model was verified against the experimental data obtained by other researchers [2, 4, 9, 10, and 11] and showed good agreement with the available experimental data as shown in Figure 4.

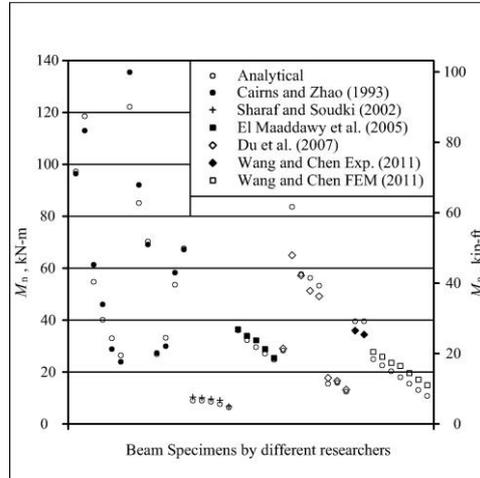


Figure 4. Comparison of analytical with Exp. results

## Effects of Different Parameters

After being validated against the available experimental data, the analytical model was employed to study the effects of the following parameters on the ultimate strength of CRC beams: corrosion rate, corrosion length  $L_{corr}$ , concrete compressive strength  $f'_c$ , length to depth ratio  $L/d$ , and loading type. Over 1000 cases were studied and it was observed that while concrete compressive strength, length to depth ratio, and loading type have minor effects on the ultimate flexural strength.

### Effect of Corrosion Length

Figure 5 shows the effects of corrosion length on the ultimate flexural strength of CRC beams of one of the cases studied. The vertical axis represents the values of  $M_{corr}/M$  for a beam with a span to depth ratio of 15. The vertical axis shows the ultimate capacity of corroded beams as a percent of the reference beam, the horizontal axis stands for the corrosion length over the total length of the beam. Each of the series represents a different corrosion rate. By investigating all the studied cases, it was noted that when the corrosion length is less than 60% of the span length, the corrosion length has no effect on the ultimate strength of CRC beams. This means that when the corrosion length is less than 60% of the span length, the reduction in ultimate flexural strength is due to the decrease in steel cross-sectional-area and strength caused by corrosion.

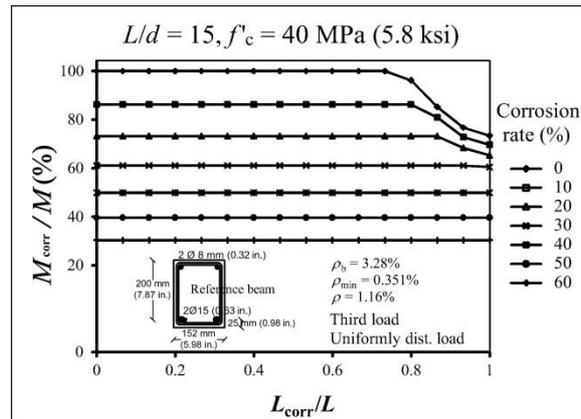


Figure 5. Effect of corrosion length on ultimate flexural strength

## Effect of Corrosion Rate

Figure 6 demonstrates the effects of corrosion rate on the ultimate flexural strength of concrete beams with corroded steel reinforcement on the tension side of the beam. While the vertical axis represents the values of  $M_{corr}/M$  for a beam with a span to depth ratio of 15, the horizontal axis shows the corrosion rate, and each of the series represents a different corrosion length. Based on over 1000 cases studied, it was concluded that the increase in corrosion rate is associated with a decrease in the ultimate flexural strength. However, when the corrosion rate is higher than 40%, the decrease in ultimate flexural strength is due to the decrease in steel cross-sectional area and strength. In other words, when the corrosion rate is higher than 40%, the corrosion length has no effect on the ultimate flexural strength.

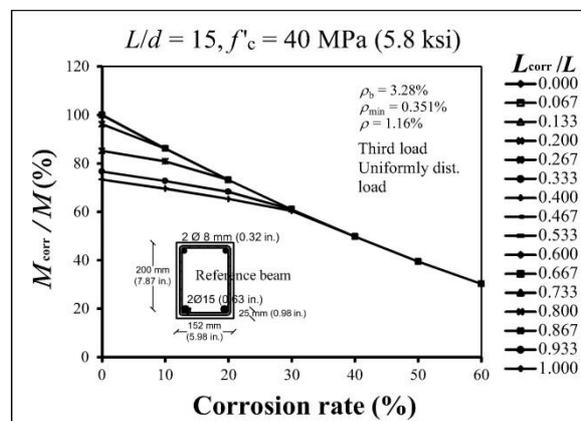


Figure 6. Effect of corrosion rate on ultimate flexural strength

## Simplified Analytical Model

### Building the Simplified Analytical Model

The detailed analytical model, after being verified against the available experimental data, was employed to study over 2000 beams that varied from structurally sound beams to unbonded and/or corroded beams under different corrosion lengths were studied. All of the cases studied were analyzed by means of  $M_{corr/ub}/M$ , which is the ratio of the ultimate moment of a beam with corroded (unbonded) reinforcement over the ultimate moment of a structurally sound beam with the same length and cross-section. Figure 7 shows some of the

cases studied where  $L/d = 15, f'_c = 40 \text{ MPa (5.8ksi)}$ .

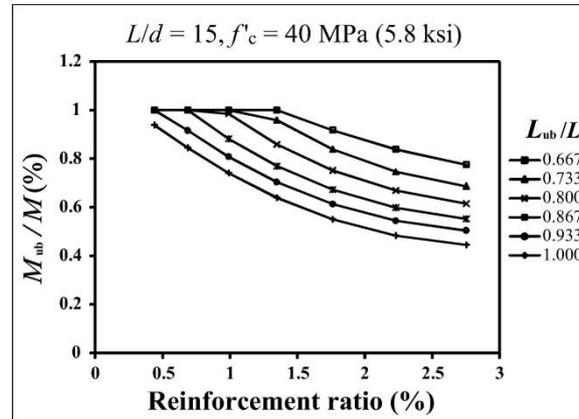


Figure 7. Ultimate capacity of RC beams with unbonded reinforcement as a percentage of structurally sound beams

Based on all the cases studied, the following equation was presented:

$$\alpha \leq \begin{cases} 1 & (\rho\% \leq 0.35) \\ 1 & \left(\frac{L_{ub}}{L} \leq 0.6\right) \\ 0.08(\rho\%)^2 - 0.56(\rho\%) + 2.25 - \frac{L_{ub}}{L} & \begin{cases} (\rho\% > 0.35) \\ \left(\frac{L_{ub}}{L} > 0.6\right) \end{cases} \end{cases} \quad (6)$$

The above equation calculates the value of the moment reduction factor  $\alpha$  based on the unbonded length and the reinforcement ratio, and the ultimate flexural strength of corroded beams can be calculated from the following equation:

$$M_{corr} = \alpha \cdot M_n^c \quad (7)$$

where  $M_n^c$  = ultimate flexural strength of a fully bonded cross section with corroded reinforcement.

$$M_n^c = C_c \cdot \left(d - \frac{\beta_1 \cdot c}{2}\right) + C_s \cdot (d - d') \quad (8)$$

where  $C_c$  = total compressive force of concrete at critical section,  $\beta_1$  = ratio of depth of rectangular stress block to depth of neutral axis,  $c$  = distance from extreme compression fiber to the neutral axis,  $C_s$  = total compressive force of compressive steel at critical section,  $d$  = distance from extreme compression fiber to centroid of tensile reinforcement,  $d'$  = distance from extreme compression fiber to centroid of compressive reinforcement.

### Validation of the Simplified Analytical Model

The results of the simplified analytical model were compared with the available experimental data obtained by other researchers [2, 4, 9, 10, and 11]. Figure 8Figure 4 shows that the simplified analytical model is able to predict the ultimate capacity of CRC beams with good accuracy.

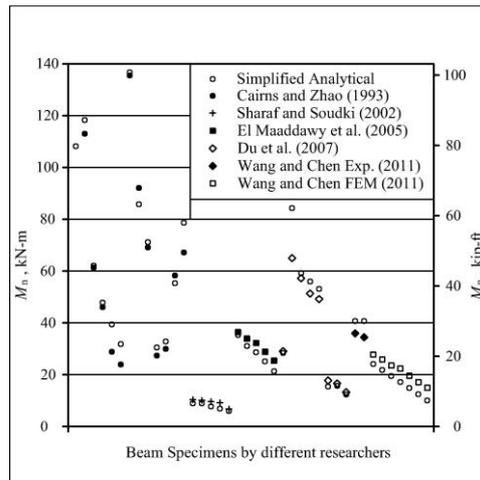


Figure 8. Comparison of simplified analytical model with the experimental results

### Summary and Conclusions

- The FEA model, detailed analytical model, and simplified analytical model are able to predict the ultimate flexural strength of CRC beams.
- When the corrosion rate is higher than 40%, the corrosion length has no effect on the ultimate flexural strength.
- When the corrosion length is less than 60%, it has no effect on the ultimate flexural strength.
- When the corrosion rate is higher than 40% or the corrosion length is less than 60%, the reduction in the ultimate flexural strength is caused by the decrease in steel cross-sectional area and strength due to corrosion.

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